Modelling of the effect of sanding on the wheel-rail adhesion area

Raphael PFAFF¹, Amir MOSHIRI¹, Alexander REICH², Markus GÄBEL²

¹Department of Mechanical Engineering and Mechatronics, Aachen University of Applied Sciences, Aachen 52064, Germany
²NOWE GmbH, Elze 31008, Germany
*Corresponding author email: pfaff@fh-aachen.de

Abstract: Modelling of the wheel-rail contact area is of interest almost since the beginning of railway operation. Current techniques, such as FASTSIM or CONTACT are typically used for analysis of running stability of rail vehicles, while other approaches such as Finite Element Analysis are applied to simulate stresses on the wheel surface. Existing techniques mostly assume limited slip and homogenous materials of wheel and rail in the contact area. These assumptions make them unsuitable for simulation of the contact patch under sanding application, since this is mostly done in high creep state and introduces a third body into the wheel-rail contact, violating the homogeneity of the materials. To gain a model of the sanding case, these assumptions are dropped and a suitable model is developed. It applies a discretisation of the Hertzian contact area as well as a stochastic model of the sand particles introduced.

Keywords: wheel-rail contact; adhesion enhancer; analytical modelling

1 Introduction

Modelling of the wheel-rail contact is a topic in railway research almost since beginning of operation. Initially, the focus was mainly on effects in the running dynamics of vehicles. Analytical modelling approaches, such as Carter, de Pater and Johnson or Haines and Ollerton dominated the research of the first decades. In contrast to these, Kalker devised the well-known CONTACT and later FASTSIM algorithm, which were developed with computational efficiency in mind. Especially FASTSIM is well used to day. More recent approaches, thanks to the advent of accessible computer power, use a finite element analysis of the wheel-rail contact area.

1.1 Review of existing approaches

1.1.1 Analytical approaches
Starting with the analysis of Hertz (Hertz, 1882), who already noted that the elastic contact of two isotropic bodies on a small area is of practical interest, the analysis of contact areas received growing attention in the mechanical engineering community. The initial works referring to Hertzian theory were dealing on fatigue of e.g. bearings. Due to the lack of appropriate computing power, the approaches remained analytical for almost a century. A first observation on the split between adhesion and slip in the contact area between wheel and rail is (Carter, 1926). Here, following the argument that wear of wheel and rail leads to a contact problem of two coaxial cylinders, an indication of a the split between adhesion and slip area is given. A more general approach was chosen in (Johnson, 1958), where two spheres are put into contact, mainly from a perspective of connecting these bodies. To further analyse the rolling contact of two bodies with friction, string theory was introduced (Haines and Ollerton, 1963) and remained a field under active development throughout the 1960s.
Prior to this, the focus of railway research shifted from the interest in traction to running stability, a field of increasing interest with the advent of high-speed rail. Here, the works of Kalker may be considered to be dominating thanks to their computational efficiency. In Kalker’s works, a distinction is made between exact theories (Kalker, 1979), i.e. those that derive their constitutive relations from elasticity theory, and simplified theories, i.e. those applying a linearization as in (Kalker, 1982). The way from exact to simplified theory was taken due to the comparably high computational load posed by the exact theory, which was prohibitive to be executed during simulation of train lateral dynamics.

The initial focus on train dynamics applications also explains the assumptions commonly made in the development of the theories:

A1 The surfaces of both bodies are smooth.
A2 Displacements are small.
A3 Creep is negligible.
A4 Both bodies behave fully elastic.
A5 The materials of both bodies are homogenous and isotropic.
A6 The dimensions of the contact patches are small compared to the main radii (i.e. wheel and rail radius).
A7 The velocity in one direction is large compared to the other components.
A8 Transient effects can be neglected.
A9 The vehicle velocity is small compared to the wave propagation velocity.
A10 The assumption of elastic contact is appropriate for the contact problem.

An overview of the assumptions made by the individual theories is shown in Table 1. It becomes evident that the theories currently applied are not readily applicable to the problem of sanding, which is mostly executed during high creep traction or braking states, so especially A3 cannot be assumed.

### Table 1 Assumptions of selected theories of wheel-rail contact

<table>
<thead>
<tr>
<th>Assumption</th>
<th>Johnson and Vermeulen (Johnson, 1958)</th>
<th>Kalker’s exact theory (Kalker, 1979)</th>
<th>Kalker’s simplified theory (Kalker, 1982)</th>
<th>Shen, Hedrick and Elkins, 1983</th>
<th>Tomberger et al., 2011</th>
<th>Model proposed in this paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>A2</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>A4</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>A5</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>A6</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>A7</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>A8</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>A9</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>A10</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

1.1.2 Empirical approaches

While the current work focuses on a white-box modeling approach, due to the predictive capabilities of such a model and the insight into the processes to be gained, in literature mostly empirical approaches were chosen to be applied especially for the high creep case. The general form, as in (Curtius and Knifler, 1950) of the model is

$$\mu_T = k_1 + \frac{k_2}{k_3 + v}$$

with the parameters as in Table 2.
Table 2 Parameters of friction coefficient curves based on empirical approach.

<table>
<thead>
<tr>
<th>Author</th>
<th>k1</th>
<th>k2/(km/h)</th>
<th>k3/(km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curtius and Kniffli</td>
<td>0.161</td>
<td>7.5</td>
<td>44</td>
</tr>
<tr>
<td>Kother</td>
<td>0.166</td>
<td>9</td>
<td>42</td>
</tr>
<tr>
<td>Parodi and Tetrel: G_a = 150 ... 200 kN</td>
<td>0</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>Parodi and Tetrel: G_a = 150 ... 200 kN</td>
<td>0</td>
<td>40</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 3 Parameters of friction coefficient curves based on empirical approach.

<table>
<thead>
<tr>
<th>Condition</th>
<th>$\mu_{bas}$</th>
<th>$\mu_0$</th>
<th>$V_{so}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wet</td>
<td>0.035</td>
<td>0.0861</td>
<td>13.94</td>
</tr>
<tr>
<td>Slippery</td>
<td>0.030</td>
<td>0.0861</td>
<td>13.94</td>
</tr>
<tr>
<td>Dry, sanded</td>
<td>0.060</td>
<td>0.1816</td>
<td>24.14</td>
</tr>
</tbody>
</table>

While these empirical results are in some form applied still today, it becomes obvious that for the increasing competitiveness of the transport modes, an appropriate modelling basis for optimization is required.

2 Modelling of the contact area

The analysis of the behaviour of the wheel-rail contact area under sanding requires information on the vertical forces acting on the wheel rail contact area, the so called normal contact problem, as well as the tangential forces transferred between wheel and rail, termed the tangential contact problem.

2.1 Geometry of the contact area

In a typical wheel-rail contact situation, i.e. the non-displaced driving surface contact, the contact between wheel and rail is dominated by the radius of the wheel, typically $r_W = (300 ... 625)$ mm and the head radius of the rail, for a UIC 60 profile $r_H = 300$ mm. Other radii, e.g. the non-constant conicity of the wheel, are large compared to these and can be assumed infinite. The contact partners are illustrated in Figure 2.

An overview of these and other national formulae for the friction coefficient in the wheel-rail-contact are presented in (Wende, 2013).

Fig. 1 Friction coefficient-velocity curve of empirical models

While these models are developed for dry rails, Metzkow (Wende, 2013) developed an empirical model based on extensive measurements on various rail conditions:

$$\mu_v = \mu_{bas} + \mu_0 \exp\left(\frac{V}{V_{so}}\right)$$

where $\mu_0$ denotes the sliding friction after wheel slide occurs.

The parameters for wet, slippery and sanded rail conditions are given in Table 3.

Fig. 2 Illustration of the geometrical situation
2.2 Normal contact mechanics

According to Hertzian theory, the resulting contact area in the geometrical setting defined above is an ellipse with a non-constant pressure \( p \) and semi-axes \( a \) and \( b \). The pressure in a coordinate system relative to the contact area is determined by

\[
p = p(x, y) = p_{\text{max}} \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}
\]

and

\[
p_{\text{max}} = \frac{3}{2\pi ab} Q
\]

where \( Q \) defines the wheel load. The semi-axes \( a \) and \( b \) can be calculated following e.g. (Iwnicki, 2006; Schindler, 2014).

This symmetric distribution assumes no damping effects of the deformed wheel or rail, i.e. perfect elastic contact. However, upon rolling of the wheel over the rail, the deformation is not perfectly elastic and shows damping between compression and decompression, which becomes evident in the rolling resistance of rail vehicles. The according pressure distribution can be obtained approximately by multiplying the local pressures \( p(x, y) \) with a linear function \( f(x) \) satisfying

\[
\int_{-a}^{a} f(x) dx = 1
\]

The according normal contact pressure distribution under assumption of the hysteresis becomes asymmetric, as is depicted in Figure 3, compared to Hertzian symmetric solution.

2.3 Tangential contact mechanics

In tangential direction, the initial main focus was on the longitudinal direction due to the aim to model the high creep traction and braking situation. It was necessary to apply a discretisation on the contact area, as in later stages, friction enhancer (e.g. sand grains) were to be introduced.

For this reason, a rectangular discretisation of the contact area was chosen, as is depicted in Figure 4 with some friction enhancer particles.

![Fig. 4 Discretisation of the contact area with friction enhancer particles](image)

For each of the rectangular discrete elements, a column, comparable to the brushes in the well-known brush model (Schindler, 2014), is assumed. The length \( l_0 \) of these columns is chosen such that for the relative error due to the deformation in \( z \)-direction (resulting in a deformed length \( l \))

\[
\varepsilon_{rel} = \frac{1}{l} - \frac{1}{l_0} \leq 0.1
\]

holds (Sextro, 2002).

These columns are deformed from the point of entering the rail towards the exit of the wheel-rail-contact due to the applied brake or traction torque. The contact area of each column can transfer a limited force, resulting from the deformation which may lead to slip occurring in the contact patch.

![Fig. 3 Asymmetric pressure distribution](image)
In this work, linear behaviour of the elastic elements in contact is assumed, therefore the strain on the individual column rises linearly from entering the contact area until this line intersects with the pressure curve due to the Hertzian contact. This intersection marks the transfer from adhesion to slip, after the intersection the individual column exhibits relaxation along the maximum transferrable force due to friction, as depicted in Figure 6.

The limiting force, defining the initiation of local slip, is defined by either the friction force, originating from the pressure in the wheel-rail-contact

\[ F = F(x, y) = \mu p(x, y) \]

or the transferrable force via a third body, e.g. a sand grain

\[ F = F_S(x, y) = \min\left(F_{\text{max, Sand}}, F_{\text{req}}(x, y)\right) \]

where \( F_{\text{req}}(x, y) \) denotes the required force in the discrete element and \( F_{\text{max, Sand}} \) the maximum transferrable force of a friction enhancer particle.

### 3 Results

#### 3.1 Results in the absence of friction enhancer

While the model was developed with the introduction of friction enhancer in mind, for sake of verification, test runs were executed without friction enhancer. These test runs, as depicted in Figure 7, exhibit a continuous behaviour of the adhesion area, as expected analytically. Furthermore, the qualitative results appear to be sensible due to the smooth introduction into the contact area and the shape of the slip area, which is congruent with published results, e.g. in (Kalker, 2013).

As further means of verification, the adhesion area can be compared to existing approaches. In this area, Kalker’s exact theory, implemented in CONTACT (Vollebregt, 2016), is considered state of the art. Figure 8 shows the adhesion area returned by CONTACT under assumption of the exact theory for parameters as in Figure 7. Friction coefficients not plotted are deemed impossible by CONTACT, as

\[ F_{\text{frict}} < \mu Q \]

is not satisfied. Obviously, for the high creep cases as discussed, the simulation of such cases is necessary.
The developed algorithm can be found to be in good accordance, however CONTACT yields a slightly different shape and size of the adhesion area. This may be caused by a different distribution of the traction or braking force along the y-axis and calls for further analysis of the software.

### 3.3 Results with friction enhancer

Forming an initial investigation into the feasibility of the approach to encompass discrete particles on the wheel-rail interface, stochastically distributed sand grains are introduced. Each grain is assumed to have a fixed maximum shear force $F_{\text{max},\text{Sand}}$, typically satisfying

$$F_{\text{max},\text{Sand}} \geq \mu p(x,y)$$

Such locally discontinuous transferrable forces would lead to a spike in transferred tangential force and deformation in $x$-direction, not in line with the anticipated mechanical behaviour. Since the stochastic distribution of the sand particles at the same time calls for a Monte Carlo-approach to predict the overall friction coefficient, a computationally efficient solution was adopted, namely smoothing of the contact area using a Gaussian filter. The Gaussian filter can be adapted to e.g. have a similar aspect ratio as the resulting Hertzian contact.

The result of a simulation of the same locomotive wheel with the introduction of friction enhancer is shown in Figure 9. In this figure, the extension of the adhesion area thanks to the presence of sand particles in the slip area becomes obvious. This partly explains the increase in friction coefficient. Further some sand particles in this instantaneous impression have no visible effect in enlarging the adhesion area, it can therefore be assumed that only sand particles active in the slip region are effective.

This led to the study of the effect of sanding under different creep conditions, as is shown in Figure 10. The authors assume that based on the current creep situation, different amounts of friction enhancers are required to effectively increase the friction coefficient. This study,
based on the developed model, may in future approaches be used to determine optimal sanding strategies.

4 Conclusions and further work

A novel model of the wheel-rail-interface, designed specifically to encompass friction enhancers and model their effects, was developed. The resulting adhesion areas are congruent to published results, however further investigation and tuning may improve this. An initial study with sand particles shows the increase in adhesion area, explaining the effect of e.g. sand on the wheel-rail interface. Further work is intended to formalize the Gaussian smoothing, implement Monte-Carlo-simulations and model the distribution of friction enhancer. It is expected that the development yields insight into a field heuristically understood, but not modelled up to now.

References


